Theoretical models of Dark Energy

Ed Copeland -- Nottingham University

1. Models of Lambda and problems
2. Scalar field models and problems

The standard cosmological models-successes and challenges

European week of Astronomy and Space Science

University of Hertfordshire -- Apr 21 2009

Some basic equations

Friedmann: $$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter.

Energy density $\rho(t)$: Pressure $p(t)$

Related through: $p = w\rho$

w=1/3 – Rad dom: w=0 – Mat dom: w = -1– Vac dom

Eqns (Λ=0):

Friedmann + Fluid conservation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

Combine

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G(\rho + 3p) --- Accn$$

$$If \;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} \quad ; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

$$\mathrm{RD} \quad : \quad w = \frac{1}{3} : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-4} \quad ; \; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$$

$$\mathrm{MD} \quad : \quad w = 0 : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3} \quad ; \; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

$$\mathrm{VD} \quad : \quad w = -1 : \rho(t) = \rho_0 \;\; ; \; a(t) \propto e^{Ht}$$

Coincidence problem – why now?

Recall: $\frac{\ddot{a}}{a} \geq 0 < - > = (\rho + 3p) \leq 0$

If: $\rho_x = \rho_x^0 \, a^{-3(1+w_x)}$

Universe dom by Quintessence at: $z_x = \left(\frac{\Omega_x}{\Omega_m}\right)^{\frac{1}{3w_x}} - 1$

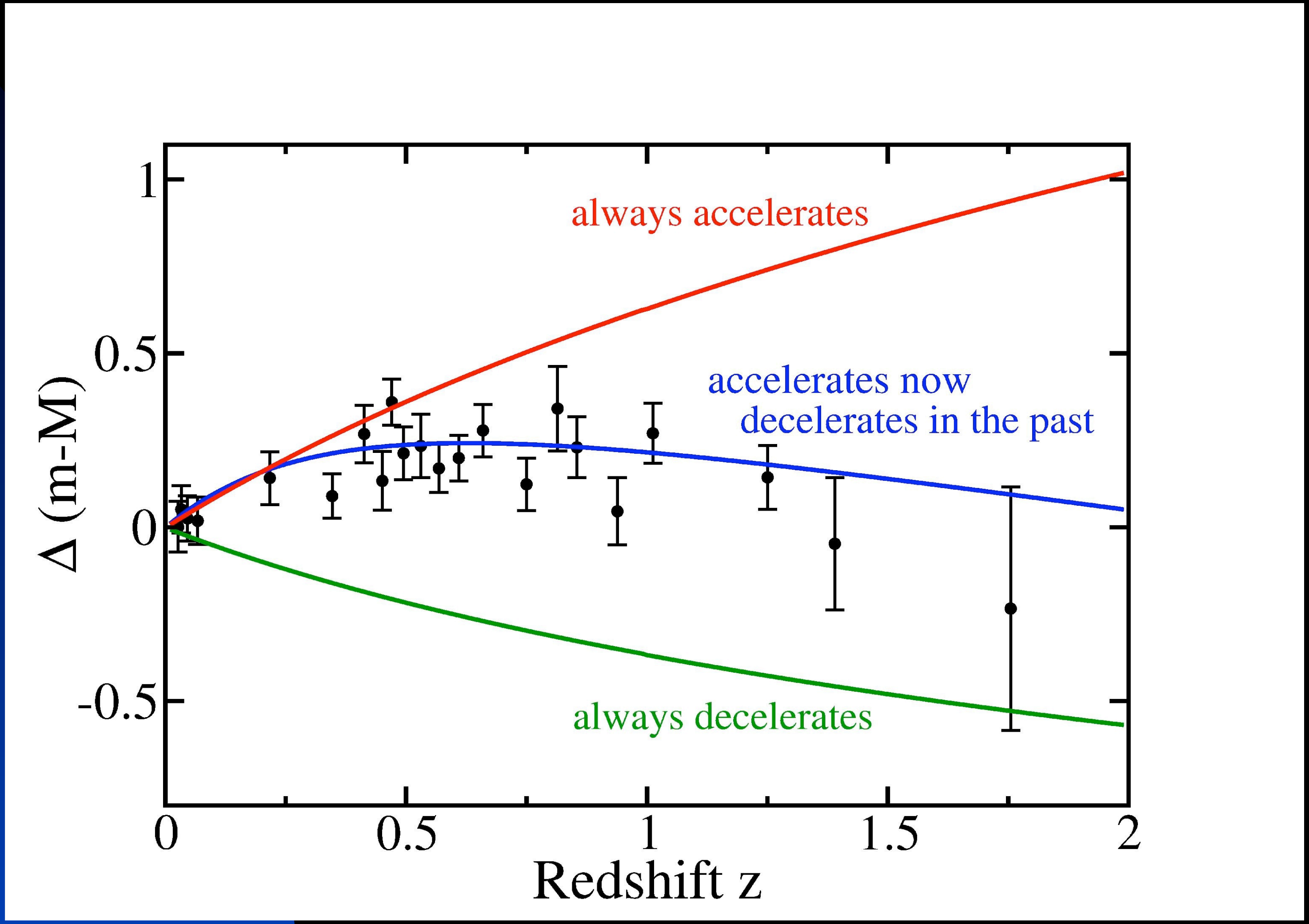
$$\left(\frac{\Omega_x}{\Omega_m}\right) = \frac{7}{3} \rightarrow z_x = 0.5,\ 0.3 \text{ for } w_x = -\frac{2}{3},\ -1$$

Univ accelerates at: $z_a = \left(-(1+3w_x)\frac{\Omega_x}{\Omega_m}\right)^{\frac{-1}{3w_x}} - 1$

$$z_a = 0.7,\ 0.5 \text{ for } w_x = -\frac{2}{3},\ -1$$

Constraint: $-0.11 < 1 + w < 0.14$ **Komatsu et al 2008 (WMAP5)**

The acceleration has not been forever -- pinning down the turnover will provide a very useful piece of information.



Different approaches to Dark Energy include amongst many:

- A true cosmological constant -- but why this value?
- Solid –dark energy such as arising from frustrated network of domain walls.
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments.
- Perhaps GR but Universe is inhomogeneous.

Over 1900 papers on archives since 1998 with dark energy in title.

Early evidence for a cosmological constant type term.

1987: Weinberg argued that anthropically ρ_{vac} could not be too large and positive otherwise galaxies and stars would not form. It should be not be very different from the mean of the values suitable for life which is positive, and he obtained $\Omega_{vac} \sim 0.6$

1990: Observations of LSS begin to kick in showing the standard $\Omega_{CDM} = 1$ struggling to fit clustering data on large scales, first through IRAS survey then through APM (Efstathiou et al)

1990: Efstathiou, Sutherland and Maddox - Nature (238) -- explicitly suggest a cosmology dominated today by cosmological constant with $\Omega_{vac} < 0.8$!

1998: Type Ia SN show evidence of cosm const.

The problem with the cosmological constant

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein (1917) -- static universe with dust

Not easy to get rid of it, once universe found to be expanding.

Anything that contributes to energy density of vacuum acts like a cosmological constant

$$< T_{\mu\nu} >=< \rho > g_{\mu\nu}$$

Lorentz inv

$$\lambda_{eff} = \lambda + 8\pi G < \rho >$$

or

$$\rho_V = \lambda_{eff}/8\pi G$$

Effective cosm const

Effective vac energy

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - < \rho > | \leq \frac{3H_0^2}{8\pi G}$$

Age Flat Non-vac matter

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - <\rho>| \leq \frac{3H_0^2}{8\pi G}$$

Hence: $\lambda_{eff} \leq H_0^2 \; or |\rho_V| \leq 10^{-29} gcm^{-3} \simeq 10^{-47} GeV^4$

Problem: expect <ρ> of empty space to be much larger. Consider summing zero-point energies (ħω/2) of all normal modes of some field of mass m up to wave number cut off Λ>>m:

$$<\rho> = \int_0^{\Lambda} \frac{4\pi k^2 dk}{2(2\pi)^3}\sqrt{k^2+m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$<\rho> = \frac{1}{2}\sum_{\text{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2+m^2}\,\frac{d^3k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where g_i are the dof of the field (+ for bosons, - for fermions).

Imagine just one field contributed an energy density $\rho_{cr} \sim (10^{-3}\,\text{eV})^4$. Implies the cut-off scale Λ<0.01 eV -- well below scales we understand the physics of.

Planck scale: $\Lambda \simeq (8\pi G)^{-1/2} \rightarrow <\rho> \simeq 2\times 10^{71} GeV^4$

But: $|\rho_V| = |<\rho> + \lambda/8\pi G| \leq 2\times 10^{-47} GeV^4$

Must cancel to better than 118 decimal places.

Even at QCD scale require 41 decimal places!

Very unlikely a classical contribution to the vacuum energy density will cancel this quantum contribution to such high precision

Not all is lost -- what if there is a symmetry present to reduce it? Supersymmetry does that. Every boson has an equal mass SUSY fermion partner and vice-versa, so their contributions to $<\rho>$ cancel.

However, SUSY seems broken today - no SUSY partners have been observed, so they must be much heavier than their standard model partners. If SUSY broken at scale M, expect $<\rho> \sim M^4$ because of breakdown of cancellations. Current bounds suggest M~1TeV which leads to a discrepancy of 60 orders of magnitude as opposed to 118 !

Still a problem of course -- is there some unknown mechanism perhaps from quantum gravity that will make the vacuum energy vanish ?

A few issues over the cosmological constant:

Is the observed dark energy really representing the energy of the vacuum or is it just that we have not yet reached it and it is a dynamical process?

The cosmological constant is the simplest addition, requires nothing other than one more fundamental constant and requires no modification of GR or addition of new fields.

How does it relate to early universe inflation? That lasted a finite time, perhaps this will imply there is nothing special about our vacuum.

Maldacena has shown stable QG vacuum of negative vacuum energy can exist (AdS/CFT), as can vacuum of zero energy (include SUSY). No one has shown a stable positive vacuum energy is possible in theories of QG. [Witten 2008]

This would imply our Universe is unstable - perhaps a bit drastic!

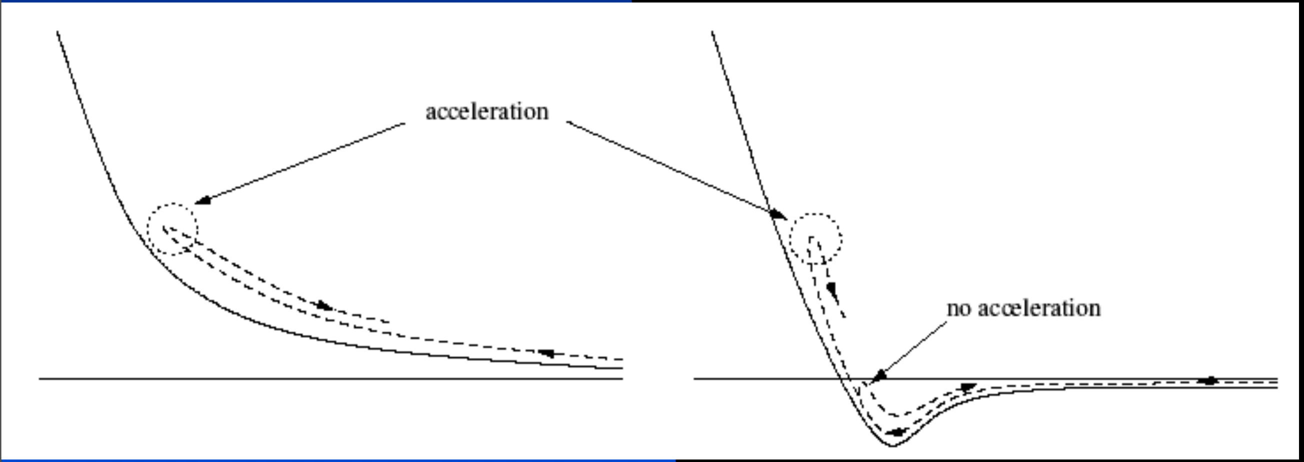
Quintessence and M-theory -- where are the realistic models?

`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

Recent extension: forbids four dimensional cosmic acceleration in cosmological solutions arising from warped dimensional reduction --[Wesley 08]

Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent, analogue of time-dependent scalar fields (radion)



Current realistic potentials are too steep

Models kinetic, not matter domination before entering accelerated phase.

Four form Flux and the cosm const: [Bousso and Polchinski]

Effective 4D theory from $M^4 x S^7$ compactification

$$S = \int d^4x \sqrt{-g}\left(\frac{1}{2\kappa^2}R + \Lambda_b - \frac{1}{2\cdot 4!}F_4^2\right)$$

Negative bare cosm const: $-\Lambda_b$

EOM: $\nabla_\mu\left(\sqrt{-g}F^{\mu\nu\rho\sigma}\right) = 0$ $\rightarrow F^{\mu\nu\rho\sigma} = c\epsilon^{\mu\nu\rho\sigma}$

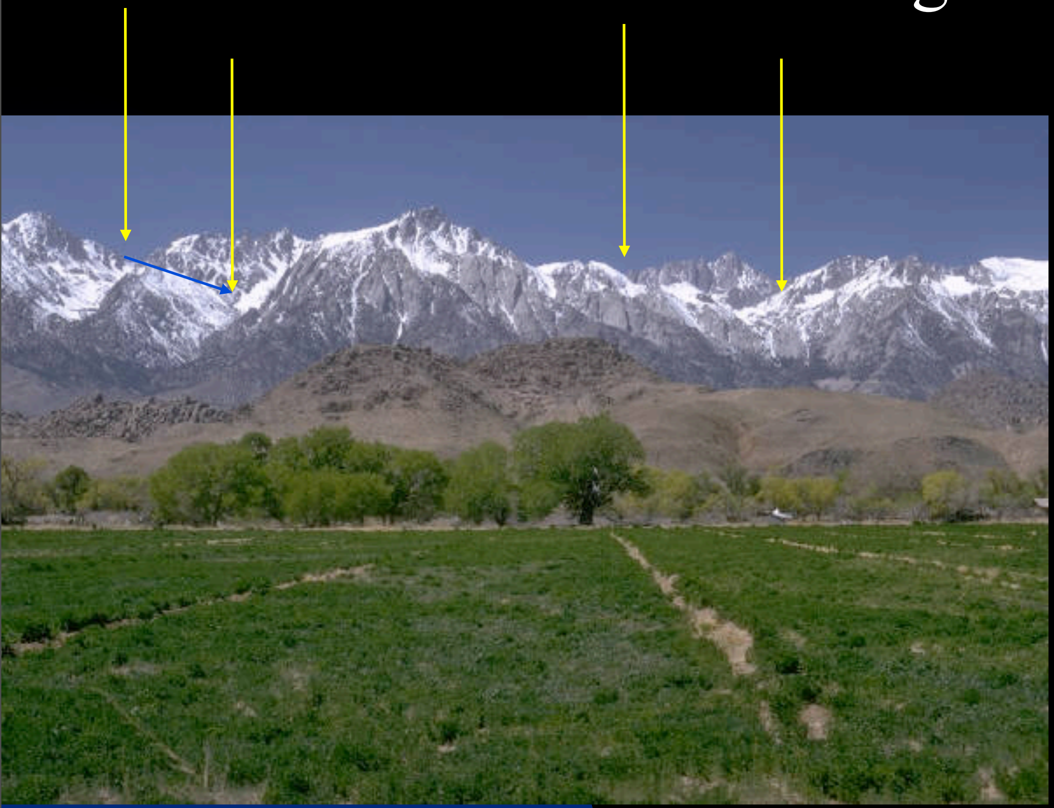
Eff cosm const: $\Lambda = -\Lambda_b - \frac{1}{48}F_4^2 = -\Lambda_b + \frac{c^2}{2}$

Quantising c and considering J fluxes $\Lambda = -\Lambda_b + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2$

Observed cosm const with J~100

Still needed to stabilise moduli but opened up way of obtaining many de Sitter vacua using fluxes -- String Landscape in which all the vacua would be explored because of eternal inflation.

1.The String Landscape approach



Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes.

Many many vacua ~ 10^{500} !

Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

Some Landscape predictions

1. Most likely our local universe born in tunnelling event from neighbouring vacuum leading to open FRW with small negative spatial curvature $\Omega_k<0$ [Freivogel et al 05] .

2. Including dynamics on probability distribution of landscape vacua. Starting from generic initial conditions, most fluxes are dynamically driven to different and narrower range of values than expected from landscape statistics alone. [Bousso and Yang (2007)]

In particular they argue cosmological evolution accesses a tiny fraction of vacua with a small cosmological constant.

[Witten 2008]

Landscape gives a realisation of the multiverse picture.

There isn't one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

This sounds like bad news, we will rely on anthropic arguments to explain it through introducing the correct measures and establishing peaks in probability distributions.

Or perhaps, it isn't a cosmological constant, but a new field such as Quintessence which will eventually drive us to a unique vacuum with zero vacuum energy -- that too has problems, such as fifth force constraints, as we will see.

For a critique of interpreting and using multiverse see talk by George Ellis at Emmanuel College Nov 07

For a defence of the Landscape and its predictive power see Polchinski - hep/th/0603249

2. Λ from a self-tuning universe [Feng et al 2001].

Λ relaxes through nucleation of branes coupled to gauge potential, the particular branes depending on the compactification assumed.

3. Relaxation of Λ [Kachru et al 2000, Arkani Hamad et al 2000, Burgess et al].

Relies on presence of extra dimension to remove the gravitational effect of the vacuum energy.

3 brane solns in 5D eff theories leads to standard model vacuum energy warping the higher dimensional spacetime while preserving 4D flatness with no cosm constant.

4. Λ from the Cyclic Perspective [Steinhardt and Turok 2002, 2006].

Key feature, because many cycles and each cycle lasts a trillion years, universe today is much older than today's Hubble time, so Λ has had long time to reduce to the observed value today.

5. Anthropic selection of Λ [Weinberg, Linde, Vilenkin, Efstathiou …].

Weinberg pointed out that once Λ dominates energy density, structure formation stops because density perturbations cease to grow. Need structure formation to complete before this otherwise no observers today. Leads to $\rho_\Lambda < 500\rho_m^{(0)}$

Two orders of magnitude out.

What if Λ differs in different parts of universe? [Efstathiou et al (1990) , Garriga and Vilenkin (2000)].

Intro conditional prob density $\mathrm{d}\mathcal{P}(\rho_\Lambda) = \mathcal{P}_*(\rho_\Lambda) n_G(\rho_\Lambda)\mathrm{d}\rho_\Lambda$

$n_G(\rho_\Lambda)$ Ave number of galaxies that can form per unit vol

$\mathcal{P}_*(\rho_\Lambda)$ A Priori probability density distribution on Λ

For a flat a priori probability density distribution it has been shown that peaks around $\mathcal{P}(\rho_\Lambda)$

$$\rho_{\rm vac} \sim 8\rho_m^{(0)}$$

[Martel et al (1998)]

Two important aspects to Anthropic argument:

1. Prediction of a priori probability
2. Assuming Λ takes on diff values in diff parts of universe.

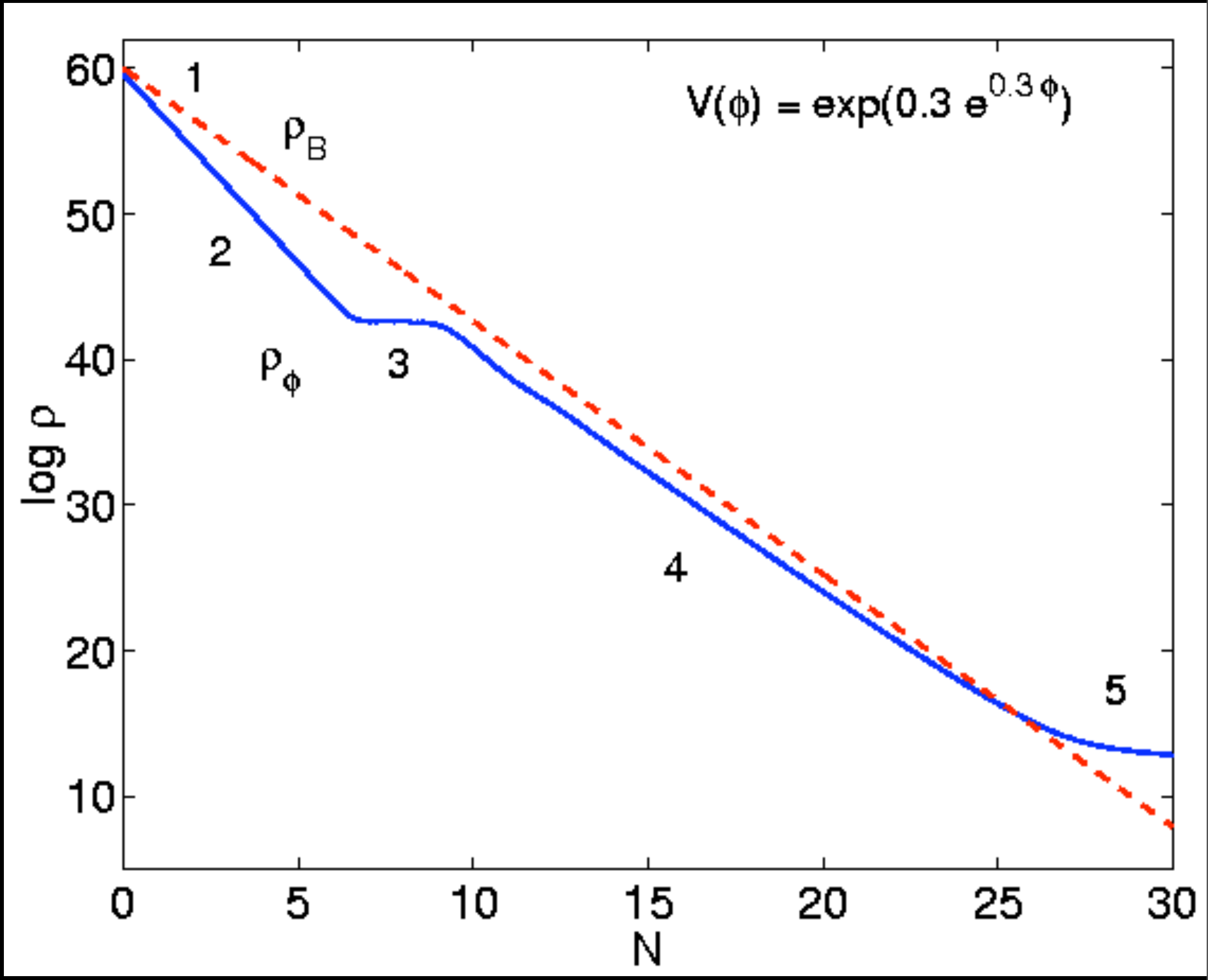
How are we going to determine the a priori probability?

See also [Garriga. et al (2005,2007), Linde (2007), Bousso et al (2007), Gibbons and Turok (2007), Easther et al (2005), Vanchurin (2007)…]

A great deal of work going on trying to determine possible measures on the multiverse and the Landscape as a manifestation of that -- no definitive conclusion yet.

Slowly rolling scalar fields
Quintessence - Generic behaviour

1. PE → KE
2. KE dom scalar field energy den.
3. Const field.
4. Attractor solution: almost const ratio KE/PE.
5. PE dom.



Nunes

Attractors make initial conditions less important

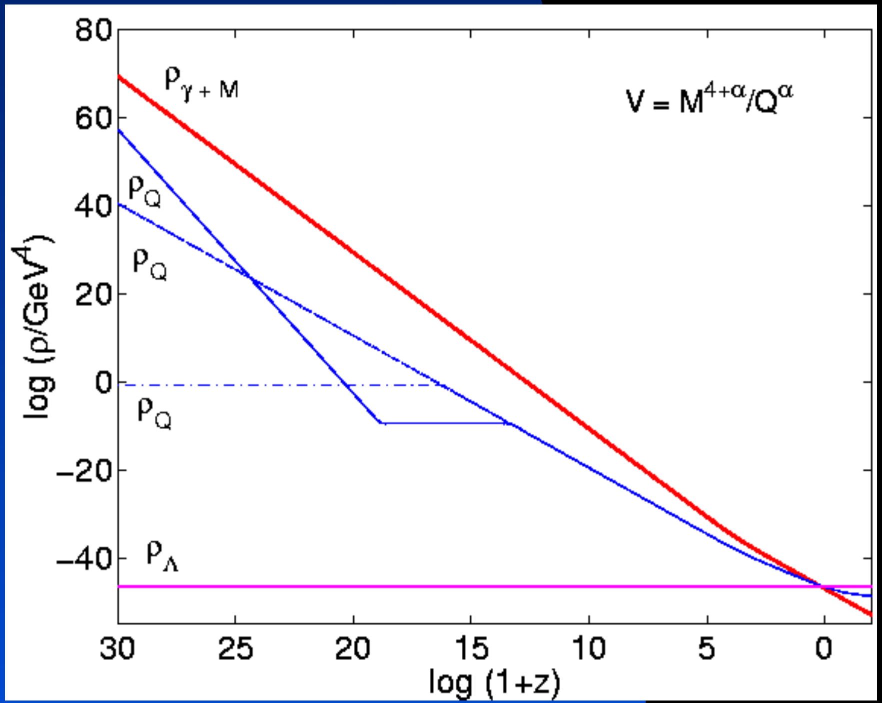
Original Quintessence model

Peebles and Ratra;

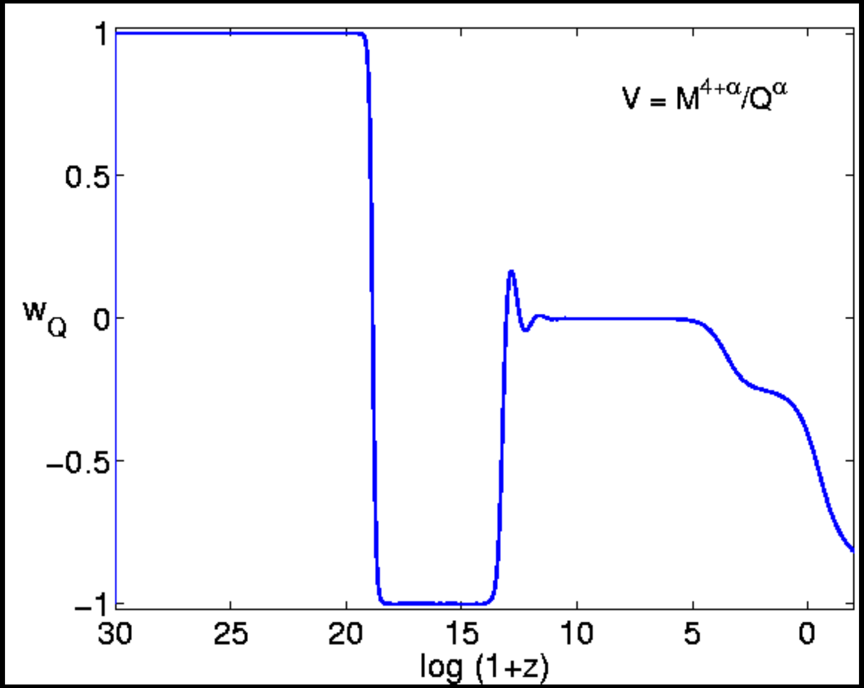
Zlatev, Wang and Steinhardt

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

Find: $\phi = \phi_i \left(\frac{a}{a_i}\right)^{\frac{3(1+w_B)}{2+\alpha}}$ and $w_\phi = \frac{\alpha w_B - 2}{2+\alpha}$



$\alpha = 6$



Fine Tuning in Quintessence

Need to match energy density in Quintessence field to current critical energy density.

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^{\alpha}}$$

$$\rho_\Lambda \leq \frac{H_0^2}{\kappa^2} \approx 10^{-47}\ \mathrm{GeV}^4$$

Find: $$y_c^2 = \frac{\kappa^2 V}{3H^2} \propto \kappa^2 \phi^2$$

so: $$H^2 = \frac{V}{\phi^2} \propto \kappa^2 \rho_\phi \Rightarrow \phi_0 \approx M_{pl}$$

Hence: $$M = \left[\rho_\phi^0\, M_{pl}^{\alpha}\right]^{1/(4+\alpha)} \Rightarrow \alpha = 2; M = 1\,GeV$$

A few models

1. Inverse polynomial – found in SUSY QCD - Binetruy

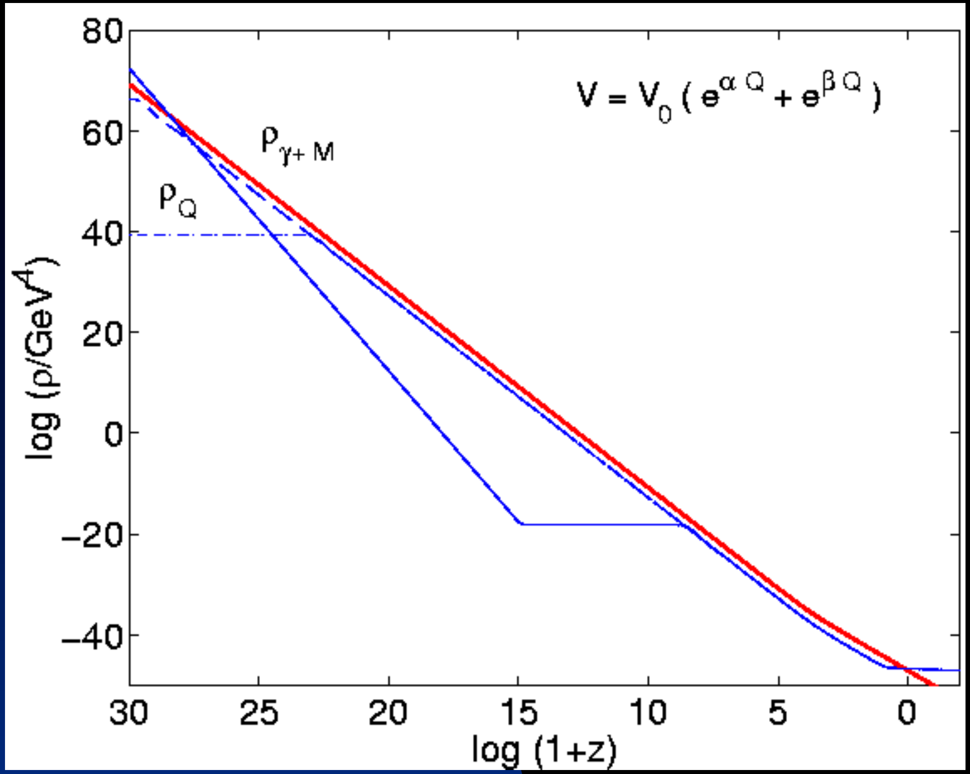
2. Multiple exponential potentials – SUGR and String compactification.

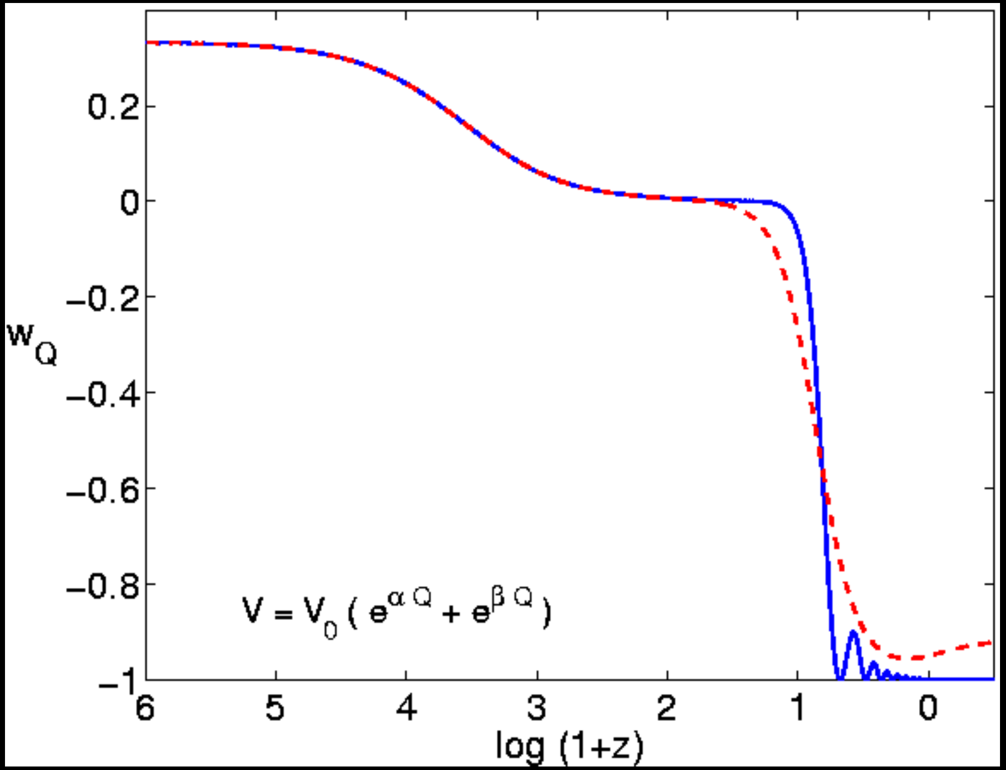
Barreiro, EC, Nunes

$$V(\phi) = V_1 + V_2$$
$$= V_{01} e^{-\kappa\lambda_1\phi} + V_{02} e^{-\kappa\lambda_2\phi}$$

Enters two scaling regimes depends on lambda, one tracking radiation and matter, second one dominating at end. Must ensure do not violate nucleosynthesis constraints.

$\alpha = 20; \beta = 0.5$





Scaling for wide range of i.c.

Fine tuning: $V_0 \approx \rho_\phi \approx 10^{-47}\ \mathrm{GeV}^4 \approx (10^{-3}\ \mathrm{eV})^4$

Mass: $m \approx \sqrt{\frac{V_0}{M_{pl}^2}} \approx 10^{-33}\ \mathrm{eV}$

Fifth force !

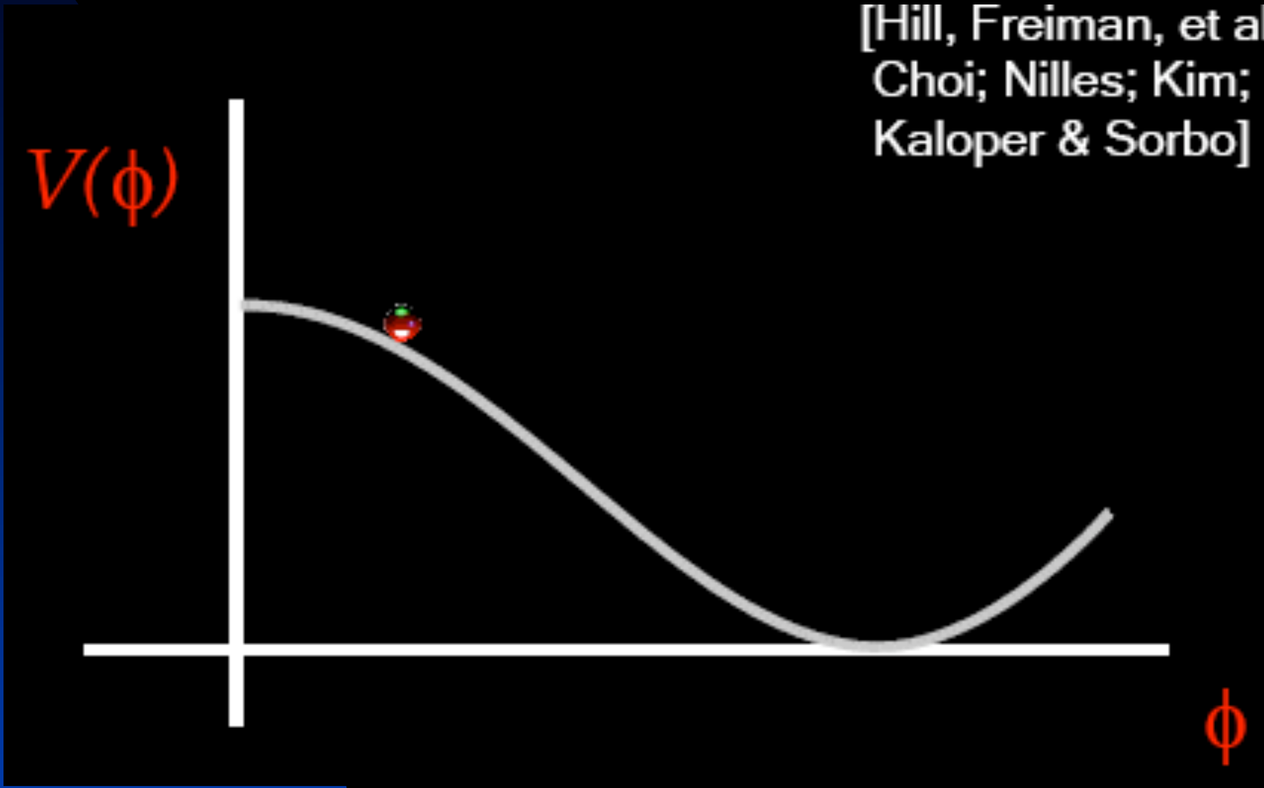
Particle physics inspired models?

Pseudo-Goldstone Bosons -- approx sym $\phi \to \phi$ + const.

Leads to naturally small masses, naturally small couplings

[Hill, Freiman, et al;
Choi; Nilles; Kim;
Kaloper & Sorbo]

Barbieri et al



$$V(\phi) = \lambda^4 (1 + \cos(\phi / F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy.

K-essence v Quintessence

K-essence -- scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model? -- Armendariz-Picon, Mukhanov, Steinhardt

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form.

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{16\pi G}R + K(\phi)\tilde{p}(X)\right]$$

$$K(\phi) > 0,\ X = \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$$

Eqn of state $$w_k = \frac{\tilde{p}(X)}{\tilde{\varepsilon}(X)} = \frac{\tilde{p}(X)}{2X\tilde{p}'(X) - \tilde{p}(X)}$$ can be < -1

However also requires similar level of find tuning as in Quintessence

Phantom fields [Caldwell (2002) …]

The data does not rule out w<-1. Can not accommodate in standard quintessence models but can by allowing negative kinetic energy for scalar field (amongst other approaches). Can arise from two time models in Type IIA strings, or low energy limit of F-theory in 12D Type IIB action.

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}(\nabla\phi)^2-V(\phi)\right]$$

leads to

$$w_\phi=\frac{p}{\rho}=\frac{\dot\phi^2+2V(\phi)}{\dot\phi^2-2V(\phi)}$$

$$w_\phi<-1 \text{ for } \dot\phi^2/2<V(\phi)$$

Super inflationary soln

$$a(t)=(t_s-t)^{\frac{2}{3(1+w)}}$$

$$H=\frac{n}{t_s-t}\,,\qquad n=-\frac{2}{3(1+w)}>0\,,$$

$$R=6\left(2H^2+\dot H\right)=\frac{6n(2n+1)}{(t_s-t)^2}\,.$$

Big Rip Singularity as $t \rightarrow t_s$

Depending on potential can avoid Big Rip but concerns over UV quantum instabilities. Vacuum unstable against production of ghosts and normal (+ve energy fields) [Carroll et al(2002), Cline et al (2004)]

Chameleon fields [Khoury and Weltman (2003) …]

Key idea: in order to avoid fifth force type constraints on Quintessence models, have a situation where the mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

Explains dark energy without violating solar system bounds.

Proposed way of detecting chameleons through Casimir Force experiments because chameleon force between two nearby bodies is more like Casimir force than gravitational force [Brax et al (07) …]

Recent suggestion: [Gies, Mota and Shaw 2007]

Afterglow as a trace of chameleon field in optical expt. Vacuum interaction of a laser pulse with B field -> production and trapping of chameleons in vacuum chamber because their mass depends on ambient matter density. Magnetically induced re-conversion of trapped chameleons into photons creates afterglow over macroscopic timescales which can be searched for in current optical expts.

Interacting Dark Energy [Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + many others…]

Idea: why not directly couple dark energy and dark matter?

$$\begin{aligned} \text{Ein eqn} &: \quad G_{\mu\nu} = 8\pi G T_{\mu\nu} \\ \text{General covariance} &: \quad \nabla_\mu G^\mu_\nu = 0 \rightarrow \nabla_\mu T^\mu_\nu = 0 \end{aligned}$$

$$T_{\mu\nu} = \sum_i T^{(i)}_{\mu\nu} \rightarrow \nabla_\mu T^{\mu\,(i)}_\nu = -\nabla_\mu T^{\mu\,(j)}_\nu \quad \text{is ok}$$

Couple dark energy and dark matter fluid in form:

$$\begin{aligned} \nabla_\mu T^{\mu\,(\phi)}_\nu &= \sqrt{\frac{2}{3}}\kappa\beta(\phi)T^{\alpha\,(m)}_\alpha \nabla_\nu\phi \\ \nabla_\mu T^{\mu\,(m)}_\nu &= -\sqrt{\frac{2}{3}}\kappa\beta(\phi)T^{\alpha\,(m)}_\alpha \nabla_\nu\phi \end{aligned}$$

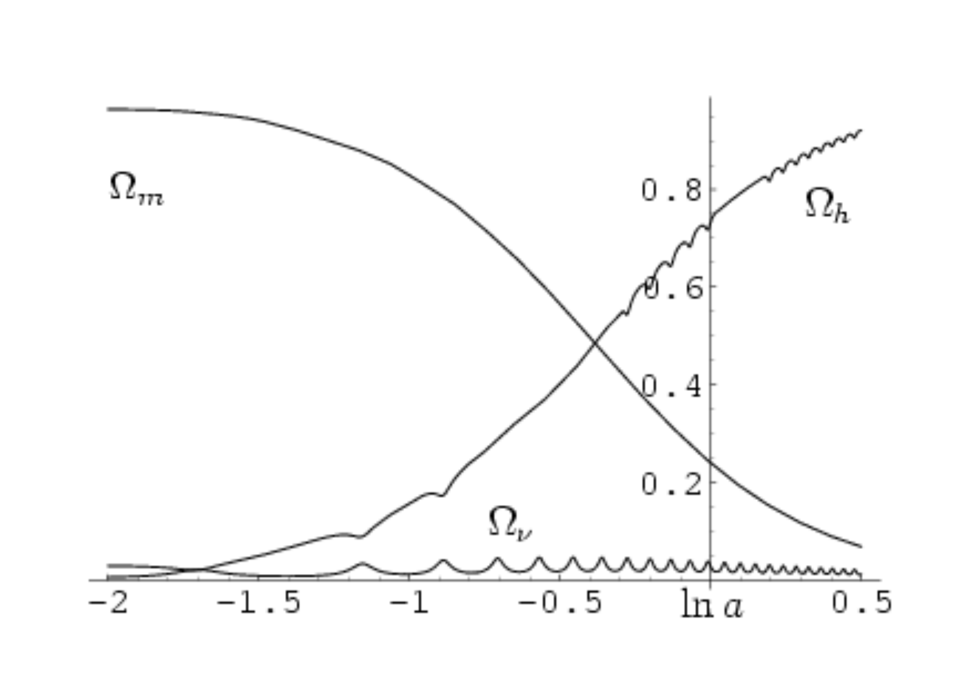
Including neutrinos -- 2 distinct DM families -- resolve coincidence problem [Amendola et al (2007)]

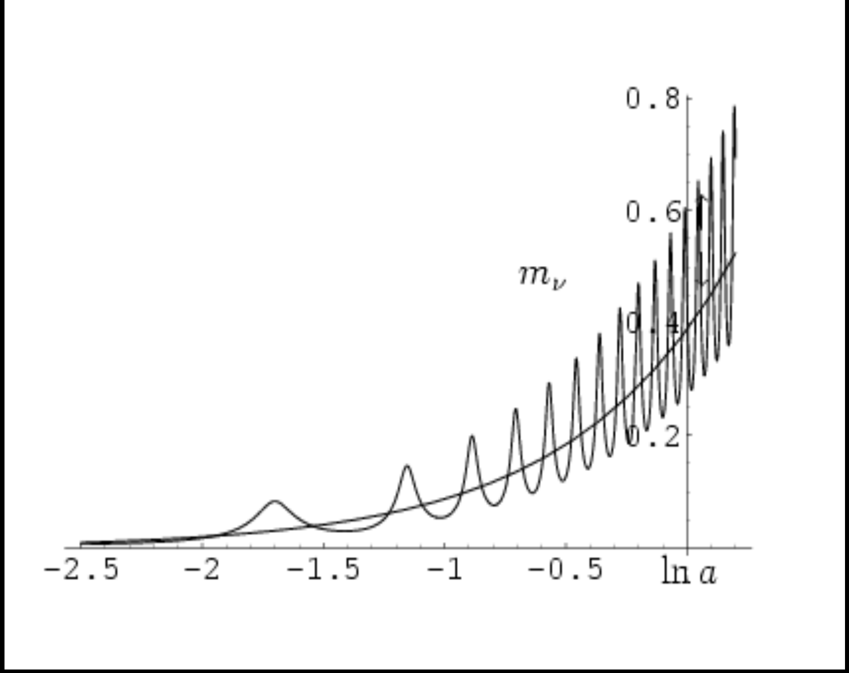
Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\mathrm{eV}}$$





m_ν

Determining the best way to test for dark energy and parameterise the dark energy equation of state is a difficult task, not least given the number of approaches that exist to modelling it .

A thorough review competed by Rocky Kolb and his colleagues making up the Dark Energy Task Force.

Albrecht et al : astro-ph/0609591

Then their findings on the search for the best figure of merit:

Albrecht et al: arXiv:0901.0721

Summary

•Observations transforming field, especially CMBR and LSS. -- everything consistent with a pure cosmological constant.

•Why is the universe inflating today?

•Is w = - 1, the cosmological constant ? If not, then what value has it?

•Is w(z) -- dynamical?

•New Gravitational Physics -- perhaps modifying Friedmann equation on large scales?

•Lots of models of dark energy but may yet prove too difficult to separate one from another such as cosmological const – need to try though!

•Perhaps we will only be able to determine it from anthropic arguments and not from fundamental theory.

•or -- could we be wrong and we do not need a lambda term?